

## PLANET-PLANET SCATTERING LEADS TO TIGHTLY PACKED PLANETARY SYSTEMS

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## ABSTRACT

The known extrasolar multiple-planet systems share a surprising dynamical attribute: they cluster just beyond the Hill stability boundary. Here we show that the planet-planet scattering model, which naturally explains the observed exoplanet eccentricity distribution, can reproduce the observed distribution of dynamical configurations. We calculated how each of our scattered systems would appear over an appropriate range of viewing geometries; as Hill stability is weakly dependent on the masses, the mass-inclination degeneracy does not significantly affect our results. We consider a wide range of initial planetary mass distributions and find that some are poor fits to the observed systems. In fact, many of our scattering experiments overproduce systems very close to the stability boundary. The distribution of dynamical configurations of two-planet systems actually may provide better discrimination between scattering models than the distribution of eccentricity. Our results imply that, at least in their inner regions which are weakly affected by gas or planetesimal disks, planetary systems should be “packed”, with no large gaps between planets.

*Subject headings:* planetary systems: formation — methods: n-body simulations

## 1. INTRODUCTION

The observed eccentricities of extra-solar planets can be readily explained by a simple model that assumes that virtually all planetary systems undergo dynamical instabilities (Ford et al. 2003; Adams & Laughlin 2003; Chatterjee et al. 2008; Juric & Tremaine 2008; Ford & Rasio 2008).<sup>8</sup> In the context of this model, planetary systems are expected to form in marginally stable configurations, meaning that they are stable for at least the timescale of rapid gas accretion of  $\sim 10^5$  years (Pollack et al. 1996) but ultimately unstable, probably on a timescale comparable to the gaseous disk’s lifetime of  $\sim 10^6$  years (Haisch et al. 2001). This instability timescale implies an initial separation between planets of perhaps 4-5 mutual Hill radii  $R_{H,M}$ , where  $R_{H,M} = 0.5(a_1 + a_2)[(M_1 + M_2)/3M_\star]^{1/3}$ ;  $a_1$  and  $a_2$  are the orbital distances,  $M_1$  and  $M_2$  are the masses of two adjacent planets, and  $M_\star$  is the stellar mass (Chambers et al. 1996; Marzari & Weidenschilling 2002; Chatterjee et al. 2008).<sup>9</sup> After a delay of  $10^5 - 10^6$  years, a typi-

cal system of three or more planets with separations of  $4 - 5R_{H,M}$  becomes unstable, leading to close encounters between two planets, strong dynamical scattering, and eventual destruction of one or two planets by either collision with another planet, collision with the star, or, most probably, hyperbolic ejection from the system (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997; Papaloizou & Terquem 2001). It is the planets that *survive* the dynamical instability that provide a match to the observed extra-solar eccentricities.

Additional dynamical information can be obtained from the known extra-solar multiple planet systems. In particular, the stability in two-planet systems can be guaranteed for planets with particular masses and orbital configurations. The edge of stability can be quantified in terms of the proximity to the analytically-derived Hill stability limit using the dimensionless quantity  $\beta/\beta_{crit}$  (the stability boundary is located at  $\beta/\beta_{crit} = 1$ ; see Section 3). Dynamical analyses have shown that the known multiple-planet systems are clustered just beyond the edge of stability (i.e., at  $\beta/\beta_{crit} \gtrsim 1$ ; Barnes & Quinn 2004; Barnes & Greenberg 2006, 2007).

In this paper we study the stability of the surviving planets in several thousand 3-planet systems that have undergone planet-planet scattering leading to the loss of one planet. We find that in the aftermath of dynamical instabilities, the surviving planets cluster just beyond the stability boundary, providing a good match to the observed values. This provides support for planet-planet scattering as an active process in extra-solar planetary systems. This result also has consequences for the packing of planetary systems and the “Packed Planetary Systems” hypothesis (Barnes & Raymond 2004; Raymond & Barnes 2005; Raymond et al. 2006; Barnes et al. 2008). The paper proceeds as follows: we describe our scattering simulations (§2), summarize Hill stability theory and define  $\beta/\beta_{crit}$  (§3), present our results (§4) and discuss the consequences (§5).

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<sup>8</sup> Several other models to explain the extra-solar eccentricity distribution exist; see Ford & Rasio (2008) for a summary.

<sup>9</sup> For Jupiter-mass planets, separations of  $\sim 4 - 5R_{H,M}$  are close to the 3:2 and 2:1 mean motion resonances. Thus, an alternate argument in favor of planets forming with such spacings invokes resonant capture (Snellgrove et al. 2001) followed by turbulent removal from resonance (Adams et al. 2008) during the gaseous disk phase.

## 2. SCATTERING SIMULATIONS

Our scattering simulations are drawn from the same sample as in Raymond et al. (2008a). Each simulation started with three planets randomly separated by 4-5 mutual Hill radii. The three planets were placed such that the outermost planet was located two (linear) Hill radii  $R_H$  interior to 10 AU ( $R_H = a[M/3M_\star]^{1/3}$ ). We performed ten sets of simulations, varying the planetary mass distribution. For our two largest sets (1000 simulations each) we randomly selected planet masses according to the observed distribution of exoplanet masses:  $dN/dM \propto M^{-1.1}$  (Butler et al. 2006). In the *Mixed1* set we restricted the planet mass  $M_p$  to be between a Saturn mass  $M_{Sat}$  and three Jupiter masses  $M_{Jup}$ . For our *Mixed2* set, the minimum planet mass was decreased to  $10 M_\oplus$ . We also performed four *Meq* sets (500 simulations each) with equal mass planets for  $M_p = 30 M_\oplus$ ,  $M_{Sat}$ ,  $M_{Jup}$ , and  $3M_{Jup}$ . Finally, the *Mgrad* sets (250 simulations each) contained radial gradients in  $M_p$ . For the JSN set, in order of increasing orbital distance,  $M_p = M_{Jup}$ ,  $M_{Sat}$ , and  $30 M_\oplus$ . For the NSJ set, these masses were reversed, i.e., the  $M_{Jup}$  planet was the most distant. The 3JJS and SJ3J sets had, in increasing radial distance,  $M_p = 3M_{Jup}$ ,  $M_{Jup}$  and  $M_{Sat}$ , and  $M_p = M_{Sat}$ ,  $M_{Jup}$  and  $3M_{Jup}$ , respectively.

Planetary orbits were given zero eccentricity and mutual inclinations of less than 1 degree. Each simulation was integrated for 100 Myr with the hybrid *Mercury* integrator (Chambers 1999) using a 20 day timestep. We required that all simulations conserve energy to better than  $dE/E < 10^{-4}$ , which is needed to accurately test for stability (Barnes & Quinn 2004). We achieved this by reducing the timestep to 5 days for simulations with  $dE/E > 10^{-4}$  and then removing simulations that still conserved energy poorly. As expected, these systems were typically unstable on  $10^5 - 10^6$  year timescales. In addition, about 1/4 of simulations were stable for 100 Myr which shows that we started close to the stability boundary. For this paper, we restrict our analysis to the subsample of simulations that 1) were unstable, and 2) contained two planets on stable orbits after 100 Myr (i.e., one and only one planet was destroyed).

## 3. HILL STABILITY

For the case of two planets with masses  $M_1$  and  $M_2$  orbiting a star, dynamical stability is guaranteed if:

$$1 + 3^{4/3} \frac{-2(M_\star + M_1 + M_2)}{G^2(M_1 M_2 + M_\star M_1 + M_\star M_2)^3} c^2 h \geq \frac{M_1 M_2}{M_\star^{2/3} (M_1 + M_2)^{4/3}} - \frac{M_1 M_2 (11M_1 + 7M_2)}{3M_\star (M_1 + M_2)^2} \quad (1)$$

where  $c$  and  $h$  represent the total orbital angular momentum and energy of the system, respectively (Marchal & Bozis 1982; Gladman 1993; Veras & Armitage 2004; note that this definition assumes that  $M_1 > M_2$ ). We refer to the left side of Eqn 1 as  $\beta$  and the right side as  $\beta_{crit}$  (Barnes & Greenberg 2006). The quantity  $\beta/\beta_{crit}$  therefore measures the proximity of a pair of orbits to the Hill stability limit of  $\beta/\beta_{crit} = 1$ . We note that our  $\beta/\beta_{crit}$  analysis only applies for two-planet non-resonant systems, because perturbations from additional companions can shift the stability boundary to values other than 1 (Barnes & Greenberg 2007).

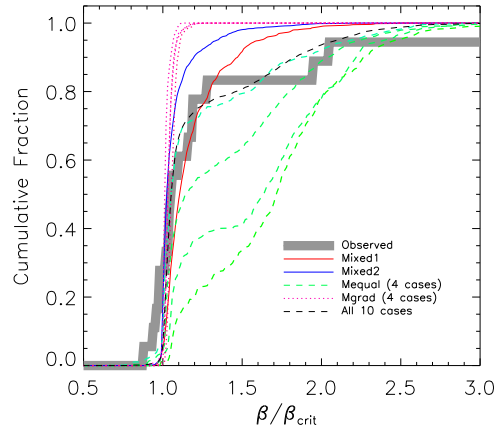


FIG. 1.— Cumulative distribution of  $\beta/\beta_{crit}$  of the well-characterized extra-solar multi-planet systems (in gray; see Table 1), as compared with our scattering simulations.

When calculating  $\beta/\beta_{crit}$  for extra-solar systems, past research (Barnes & Greenberg 2006, 2007) has assumed coplanar orbits with masses equal to minimum masses. Those values of  $\beta/\beta_{crit}$  were systematically affected by the mass-inclination degeneracy, probably resulting in overestimations. In contrast, our simulations provide the full three-dimensional orbits, and hence we can calculate the true value of  $\beta/\beta_{crit}$ . More importantly, if we assume that viewing geometries are distributed isotropically (i.e. edge-on systems are more likely than face-on), we can determine how  $\beta/\beta_{crit}$  would be calculated from radial velocity data (e.g. assuming coplanar, edge-on orbits). For example, if two planets with masses  $M_b$  and  $M_c$  have inclinations (relative to their invariable plane)  $i_b$  and  $i_c$ , and the inclination to the line of sight is  $I$ , then the “observed”  $\beta/\beta_{crit}$  would use masses  $M_b \sin(i_b + I)$  and  $M_c \sin(i_c + I)$ . In §4 we use this approach to build a distribution of  $\beta/\beta_{crit}$  that is directly comparable to the actual distribution (and effectively break the mass-inclination degeneracy).

## 4. RESULTS

We generated  $\beta/\beta_{crit}$  distributions from our simulations following the procedure described above. First, we “observed” each system from 100 viewing angles, thereby decreasing the inferred mass of each planet by a factor of  $\sin(I + i_j)$ , where  $i_j$  refers to each planet’s inclination with respect to a fiducial plane. Second, we assumed the observed systems to be coplanar in calculating  $\beta/\beta_{crit}$  for each viewing angle using Eqn 1. Finally, we included the  $\beta/\beta_{crit}$  calculated for each viewing angle by assuming the viewing angle  $I$  to be isotropically distributed. Figure 1 compares the cumulative  $\beta/\beta_{crit}$  distributions for the observed two-planet systems with our scattering simulations. It is important to note that the “true”  $\beta/\beta_{crit}$  distributions, calculated with knowledge of the simulated systems’ real masses and inclinations, are virtually identical to the curves from Fig. 1 (this issue is discussed further in §5). Table 1 lists the extra-solar systems in our analysis; we excluded systems with controversial or poorly-characterized orbits and those that were likely affected by tidal effects. In a two planet system with an inner planet at  $\lesssim 0.1$  AU, tides will decrease the inner planet’s eccentricity and semimajor axis (Jackson et

TABLE 1  
PLANETARY SYSTEMS INCLUDED IN  $\beta/\beta_{crit}$  ANALYSIS<sup>1</sup>

System (pair)	$a_1, a_2$ (AU)	$e_1, e_2$	$M_1, M_2$ ( $M_{Jup}$ )	$\beta/\beta_{crit}$
HD 202206 b-c <sup>2</sup>	0.83, 2.55	0.435, 0.267	17.4, 2.44	0.883
HD 82943 c-b <sup>2</sup>	0.746, 1.19	0.359, 0.219	2.01, 1.75	0.946
HD 128311 b-c <sup>2</sup>	1.099, 1.76	0.25, 0.17	2.18, 3.21	0.968
HD 73526 b-c <sup>2</sup>	0.66, 1.05	0.19, 0.14	2.9, 2.5	0.982
HD 45364 b-c <sup>2</sup>	0.681, 0.897	0.168, 0.097	0.187, 0.658	0.989
47 UMa b-c	2.11, 3.39	0.049, 0.22	2.6, 0.46	1.025
HD 155358 b-c	0.628, 1.224	0.112, 0.176	0.89, 0.504	1.043
HD 177830 c-b	0.514, 1.22	0.40, 0.041	0.186, 1.43	1.046
HD 60532 b-c <sup>2</sup>	0.77, 1.58	0.278, 0.038	3.15, 7.46	1.054
HD 183263 b-c	1.52, 4.25	0.38, 0.253	3.69, 3.82	1.066
HD 108874 b-c <sup>2</sup>	1.051, 2.68	0.07, 0.25	1.36, 1.018	1.10
HD 12661 b-c	0.83, 2.56	0.35, 0.2	2.3, 1.57	1.12
HD 11506 c-b	0.639, 2.43	0.42, 0.22	0.82, 3.44	1.17
HD 208487 b-c	0.49, 1.8	0.32, 0.19	0.45, 0.46	1.20
HD 169830 b-c	0.81, 3.60	0.31, 0.33	2.88, 4.04	1.28
HD 168443 b-c	0.3, 2.91	0.529, 0.212	8.02, 18.1	1.95
HD 38529 b-c	0.129, 3.68	0.29, 0.36	0.78, 12.7	2.06
HD 47186 b-c	0.05, 2.395	0.038, 0.249	0.072, 0.35	6.13

<sup>a</sup>See <http://www.astro.washington.edu/users/rory/research/xsp/dynamics/> for an up to date list of  $\beta/\beta_{crit}$  values for the known extra-solar multiple planet systems. Orbital values were retrieved from <http://exoplanet.eu> and <http://exoplanets.org>.

<sup>b</sup>These systems have been claimed to be in mean motion resonances.

al. 2008), thereby increasing the separation between the two planets and  $\beta/\beta_{crit}$ .

Four individual cases – **Mixed1**, **Mixed2**, **Meq:M<sub>sat</sub>**, and **Meq:30 M<sub>⊕</sub>** – each provide a match to the observed  $\beta/\beta_{crit}$  distribution. Kolmogorov-Smirnov (K-S) tests show that the probability  $p$  that the  $\beta/\beta_{crit}$  distributions from those four cases are drawn from the same distribution as the observed sample are all 0.1 or larger (Table 2). The distribution calculated by an unweighted combination of all ten cases is also a good match (each case was given equal weight, regardless of the number of simulations).

All of our sets of simulation produced a smaller fraction of systems at  $\beta/\beta_{crit} < 1$  than for the observed systems. We therefore calculated K-S  $p$  values by confining the distributions to the ranges  $\beta/\beta_{crit} \leq 1$  and  $\beta/\beta_{crit} > 1$ . All but one case with  $p \geq 0.1$  also had  $p(\beta/\beta_{crit}) > 0.1$  (**Mixed2**; see Table 2). However, some cases provide good matches for  $\beta/\beta_{crit} \leq 1$  but not for other regions, notably **Meq:M<sub>Jup</sub>**, **Mgrad:JSN**, and **Mgrad:NSJ**. Systems with  $\beta/\beta_{crit} < 1$  are unusual because they lie within the formal Hill stability boundary but are stabilized by special orbital configurations. In fact, all five of the known exoplanet systems with  $\beta/\beta_{crit} < 1$  are thought to lie in mean motion resonances (Table 1). The scattered systems with  $\beta/\beta_{crit} < 1$  are stabilized by resonances or in many cases by low-amplitude, aligned apsidal libration. Four cases in our sample generated resonant systems in at least 5% of simulations (Raymond et al. 2008a) – **Mixed2**, **Mgrad:JSN**, **Mgrad:NSJ**, and **Mgrad:SJ3J** – but only two of these have  $p(\beta/\beta_{crit} \leq 1) > 0.1$ . We attribute the lack of a correlation between resonances and  $\beta/\beta_{crit} < 1$  to the relative weakness of these resonances. Indeed, resonances caused by scattering tend to exhibit relatively high-amplitude libration of only one resonant argument (Raymond et al. 2008a); these resonances have typical  $\beta/\beta_{crit}$  values of slightly more than

TABLE 2  
 $p$  VALUES FROM K-S TESTS OF OBSERVATIONS VS. SCATTERING SIMULATIONS

Case	$p$	$p(\beta/\beta_{crit} \leq 1)$	$p(\beta/\beta_{crit} > 1)$
<b>Mixed1</b>	0.14	$6.2 \times 10^{-3}$	0.53
<b>Mixed2</b>	0.13	$1.2 \times 10^{-4}$	0.07
<b>Meq:3M<sub>Jup</sub></b>	$1.8 \times 10^{-6}$	$2.1 \times 10^{-5}$	$4.6 \times 10^{-4}$
<b>Meq:M<sub>Jup</sub></b>	$9.3 \times 10^{-4}$	0.10	0.02
<b>Meq:M<sub>sat</sub></b>	0.12	0.81	0.43
<b>Meq:30M<sub>⊕</sub></b>	0.29	0.60	0.89
<b>Mgrad:JSN</b>	$4.0 \times 10^{-3}$	0.12	$2.4 \times 10^{-4}$
<b>Mgrad:NSJ</b>	$1.5 \times 10^{-4}$	0.10	0.02
<b>Mgrad:3JJJ</b>	$9.1 \times 10^{-3}$	$1.9 \times 10^{-5}$	$9.8 \times 10^{-4}$
<b>Mgrad:SJ3J</b>	$4.9 \times 10^{-3}$	$6.2 \times 10^{-4}$	$2.8 \times 10^{-4}$
All 10 cases	0.14	$5.1 \times 10^{-3}$	0.58

1 (median  $\beta/\beta_{crit} = 1.01 - 1.03$  for the different cases). This contrasts with resonances generated by convergent migration in gaseous disks, which tend to exhibit low amplitude libration of more than one resonant argument (Snellgrove et al. 2001; Lee & Peale 2002).

Simulations with radial mass gradients (**Mgrad**) over-produced systems very close to the stability boundary, while cases with equal masses (**Meq**) produced much larger  $\beta/\beta_{crit}$  values (Fig. 1). A similar effect was seen in the eccentricity distributions: the **Mgrad** cases yielded much smaller eccentricities than the **Meq** cases (Raymond et al. 2008a; see also Ford et al. 2003). The **Mixed1** and **Mixed2** cases fall between these two regimes. These trends can be explained by the number of close encounters  $n_{enc}$  that occur in the different cases before a planet is destroyed. For the **Mgrad** cases  $n_{enc}$  is typically between 30 and 80, and is larger for less massive systems (**JSN** and **NSJ**). For the **Meq** cases  $n_{enc}$  is vastly larger, with median values between 100 (**3M<sub>Jup</sub>**) and 2000 (**30M<sub>⊕</sub>**). The larger number of scattering events increases the eccentricity of surviving planets and also causes the systems to spread out farther.

In calculating “observed”  $\beta/\beta_{crit}$  distributions from our simulations, we assumed that the viewing angles  $I$  were isotropically distributed. Given that known extra-solar planet systems are each observed at a fixed  $I$ , could this have introduced a bias in our samples? Figure 2 shows the inferred value of  $\beta/\beta_{crit}$  as a function of  $I$  for five **Mixed1** systems with varying mutual inclinations  $\Delta i$ . For  $\Delta i \lesssim 35^\circ$ ,  $\beta/\beta_{crit}$  varies only slightly with the viewing angle, but for large  $\Delta i$   $\beta/\beta_{crit}$  can change substantially with  $I$ .<sup>10</sup> However,  $\beta/\beta_{crit}$  varies by more than 10% [20%] over the entire range of possible viewing angles for fewer than 10% [2%] of cases. For all systems, the edge-on  $\beta/\beta_{crit}$  values agree with the true  $\beta/\beta_{crit}$  values (calculated with knowledge of the planets’ true masses and orbits) to better than 10%. There is a small bias:  $\sim 80\%$  of systems exhibit a shallow negative slope in  $\beta/\beta_{crit}$  vs.  $I$ , suggesting that the majority of inferred  $\beta/\beta_{crit}$  values may be overestimated but only by  $\lesssim 1\%$ . Thus, although  $I$  and  $\Delta i$  are important to keep in mind, they introduce a negligible error into our analysis.

## 5. DISCUSSION

<sup>10</sup> We have found that it is actually the angular momentum deficit (Laskar 1997) which controls the magnitude of  $\beta/\beta_{crit}$  variation with  $I$ .

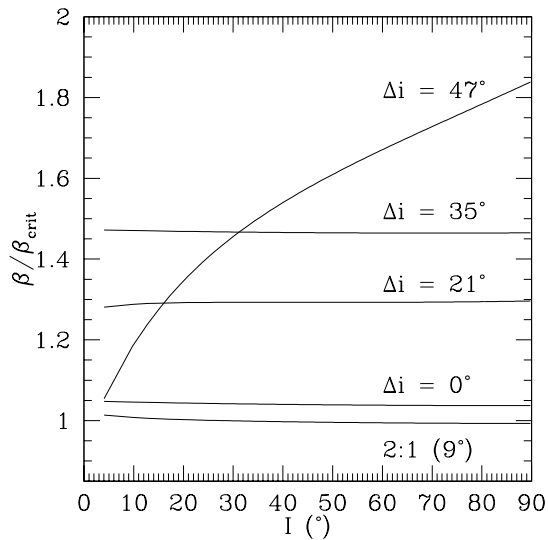


FIG. 2.— Inferred values for  $\beta/\beta_{crit}$  as a function of observation angle  $I$  for several examples from the *Mixed1* set, labeled by the approximate mutual inclination  $\Delta i$  between planets. One resonant case is labeled “2:1”: it has  $\Delta i = 9^\circ$ .  $I = 90^\circ$  is edge-on and  $I = 0^\circ$  is face-on.

The planet-planet scattering model appears to be consistent with the  $\beta/\beta_{crit}$  distribution of the observed exoplanet systems. The distribution can be reasonably reproduced by several of our sets of simulations, or even by an unweighted combination of all ten sets. We therefore cannot strongly constrain the initial planetary mass distribution, although we can rule out cases with very poor fits – *Meq:3M<sub>Jup</sub>*, *Mgrad:3JJS* and *Mgrad:SJ3J* in particular – as the major contributors to the distribution (see Table 2). We consider the *Mixed1* set to be the most realistic because it is drawn from the observed mass distribution (Butler et al. 2006), and it provides a good match to the observed eccentricity distribution (Raymond et al. 2008a). In the coming years, we expect many more systems to be discovered with  $\beta/\beta_{crit} \approx 1 - 1.5$ .

The pileup of scattered systems just beyond the stability boundary implies that planetary systems are “packed”, meaning that large spaces in between planets should be rare.<sup>11</sup> This provides a theoretical foundation for the “Packed Planetary Systems” hypothesis, which asserts that if a stable zone exists between two known planets, then that zone is likely to contain a planet (Barnes & Raymond 2004; Raymond & Barnes 2005; Raymond et al. 2006). Given the small  $\beta/\beta_{crit}$  values of scattered systems, there is simply no room to insert

another planet between the two known planets without causing the system to be unstable.

HD 74156 is an example of a packed planetary system. Prior to 2008, two planets were known in the system, at 0.28 and 3.4 AU (Naef et al. 2004), with  $\beta/\beta_{crit} = 1.987$ . Raymond & Barnes (2005) mapped out a narrow stable zone between the two planets, from 0.9–1.4 AU. The planet HD 74156 d was discovered three years later by Bean et al. (2008) at 1.01 AU (see also Barnes et al. 2008) at the peak of the stable zone. We therefore expect additional planets to exist in systems with  $\beta/\beta_{crit} > 1.5 - 2$ , notably HD 38529 (Raymond & Barnes 2005) and HD 47186 (Kopparapu et al. 2009). The probable location of additional planets can be determined using test planets to map out dynamically stable regions between known planets (e.g., Menou & Tabachnik 2003; Rivera & Haghighipour 2007; Raymond et al. 2008b).

The  $\beta/\beta_{crit}$  distribution of the observed extra-solar planetary systems may contain information about different dynamical regimes. The region of  $\beta/\beta_{crit} \leq 1$  is populated entirely by resonant systems and may provide evidence of planetary system compression, presumably via convergent migration in gaseous protoplanetary disks (Snellgrove et al. 2001; Lee & Peale 2002). The region of  $1 \leq \beta/\beta_{crit} \leq 1.5 - 2$  is consistent with the scattering regime. Widely-separated systems with  $\beta/\beta_{crit} > 1.5 - 2$  may have been drawn apart by interactions with planetesimal or gaseous disks (e.g., Gomes et al. 2004; Moeckel et al. 2008). However, this seems unlikely given that the known planets lie relatively close to their stars and that disk effects should be far more pronounced at large distances.

Given that our simulations started with only three planets, we could not calculate  $\beta/\beta_{crit}$  values in perturbed two-planet systems. For example, an interesting comparison with observations would be to measure  $\beta/\beta_{crit}$  for the two easiest-to-detect planets in scattered three planet systems. This would address the question of whether to search for additional planets in between or interior/exterior to the known planets in two planet systems with large  $\beta/\beta_{crit}$ .

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planets ranges from  $<2$  AU to  $>15$  AU.

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